Exercise 54

Find equations of the tangent lines to the curve

$$y = \frac{x-1}{x+1}$$

that are parallel to the line $x - 2y = 2$.

Solution

Solve the equation of this line for y.

$$y = \frac{1}{2}x - 1$$

The line's slope is $1/2$. The aim is to find the values of x where the given function's slope is $1/2$. Start by taking the derivative.

$$y' = \frac{d}{dx}\left(\frac{x-1}{x+1}\right) = \frac{2}{(x+1)^2}$$

Then set it equal to $1/2$ and solve the equation for x.

$$\frac{2}{(x+1)^2} = \frac{1}{2} \quad \rightarrow \quad x = \{-3, 1\}$$

Again, these values of x are where the slope on the given curve is $1/2$. Plug these values of x into the given function.

$$y(-3) = \frac{-3-1}{-3+1} = 2$$

$$y(1) = \frac{1-1}{1+1} = 0$$

The points that the (two) tangent lines touch the curve are then

$$(-3, 2) \quad \text{and} \quad (1, 0).$$

Therefore, the equations of the tangent lines are

$$y - 2 = \frac{1}{2}(x+3)$$

$$y - 0 = \frac{1}{2}(x-1).$$

These lines are plotted below with the given function.